Processing non-Newtonian products: Determining the pressure drop for a power law fluid along a straight circular pipe

RHEOLOGY AND VISCOSITY

In the chemical and process industries, it is often required to pump fluids over long distances from storage to various processing units and/or from one plant site to another. It is thus often necessary to calculate the pressure requirements for pumping, the selection of optimum pipe diameter as well as measurement and control of flow rate. Many of the formulae required to estimate such parameters are available in the literature and require some knowledge of these processing parameters as well as fluid properties.

When dealing with non-Newtonian fluids it is often sufficient to consider them as power law fluids in terms of processing due to the shear rates involved.

If the liquid obeys power law behaviour then the pressure drop across the pipe can be described by the following equation (1):

$$\Delta P = \frac{2kL}{r}\left[Q.\frac{3+1/n}{\pi r^3}\right]^n \qquad (1)$$

where *k* is the consistency and *n* the power law index; *Q* is the flow rate through the pipe radius *r* with a pressure drop ΔP. If the fluid is Newtonian then the power law index has a value of 1.
The shear rate encountered during this process is given by the following expression (2):

$$\dot{\gamma} = \frac{Q}{\pi r^3}\left[3+\frac{1}{n}\right] \qquad (2)$$

By measuring the volumetric flow rate for a given pipe diameter it is therefore possible to estimate the shear rate encountered during the pumping process. If n is unknown at this stage then it can be taken as 1, which is the value for a Newtonian fluid. Measuring the viscosity at selected shear rates slightly above and below the calculated value allows a relevant portion of the flow curve to be generated. A power law model can then be fitted to the data and values of k and n determined. These values can then be inputted in to equations 1 and 2 to yield the pressure drop across the pipe and the true shear rate respectively. These expressions assume steady state (fully developed) laminar flow and no slip conditions at the pipe walls.

Experimental

- This example considers a shampoo product being transported through a straight pipe with a radius of 0.0125m and length of 10m. The volumetric flow rate is 0.0005m^3/s and the power law index was known to be 0.15.
- Rotational rheometer measurements were made using a Kinexus rotational rheometer with a Peltier plate cartridge and a cone-plate measuring system1, and using standard pre-configured sequences in the rSpace software.
- A standard loading sequence was used to ensure that both samples were subject to a consistent and controllable loading protocol.
- All rheology measurements were performed at 25°C.
- The relevant shear rate for flow in the pipe was automatically calculated as part of the test sequence using inputted values of pipe radius, length, volumetric flow rate and power law index.
- A shear rate table using a start value of (calculated shear rate/2) and an end value of (calculated shear rate ×2) was performed, and a power law model fitted to the resultant flow curve and the calculated pressure drop determined.

Results and Discussion

From the information provided the calculated shear rate for flow in the pipe was determined to be 787 s^{-1}. This automatically generated a table of shear rates between 394 s^{-1} and 1578 s^{-1} and produced a shear thinning curve as shown in Figure 1.

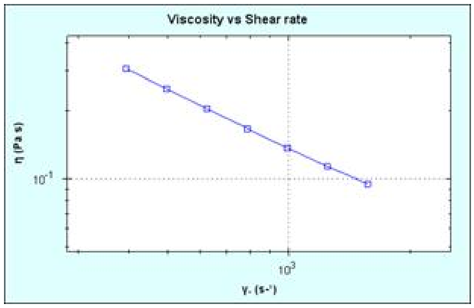

Figure 1 – Viscosity vs. shear rate plot (on log axes) for a shampoo over the calculated shear rate range

A power law analysis on the resultant curve yielded values of k and n of 48.7 and 0.1506 respectively. These values where then used to determine the true shear rate (if n was not known initially), the pressure drop and the associated shear stress.

These calculated values were then displayed as a prompt in the rSpace software as shown in Figure 2.

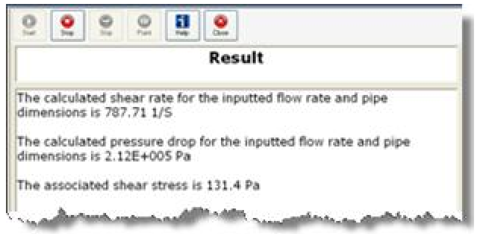

Figure 2 – Calculated values for pressure drop, shear rate and shear stress displayed as a prompt

To pump this material at the required flow rate will therefore require a pressure difference across the pipe of 212 kPa and an associated shear stress of 131.4 Pa

Conclusion

A shear rate value was calculated from input values of flow rate and pipe dimensions, which were used to generate a flow curve. Equation 1 was then used to determine the pressure drop across the pipe based on parameters obtained from a power law analysis of the curve. This sequence is therefore useful for predicting pressure requirements for achieving the required flow rate in a straight circular pipe.

References

A Handbook of Elementary Rheology; HA Barnes

Non-Newtonian Flow in the Process Industries; RP Chaabra & JF Richardson

- Note that testing is recommended to be undertaken with cone and plate or parallel plate geometry – with the latter being preferred for dispersions and emulsions with large particle sizes. Such material types may also require the use of serrated or roughened geometries to avoid artefacts relating to slippage at the geometry surface.

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